

Appendix 5.2 Manual calculation of trial vectors

Here, the manual arithmetical calculation of a trial vector shall be demonstrated to understand how they are arrived at. This is considered important to engage with, since it would be pernicious to suggest something about how the original grids are analysed in the context of the natural kinds concepts, without an inkling of what is being done to the original data.

The method performed here until the calculation of the trial vectors is based on the original method of Hotelling (1933) and described by Kline (2002). The relevance of this section to the discussion on natural kinds is the role of *iteration* in this protocol, which is central to PCA (Kline, 2002).

Step 1: Sum the coefficients in each column, U_{a1}

Step 2: Normalise U_{a1}

In Table 1., the columns were summed and squared. The sums squared were summed in turn ($\Sigma = 85.828$) and square rooted ($= 9.264$). Each element was divided by this square root to provide a normalised U_{a1} called V_{a1} .

Table 1. Sums of the columns, U_{a1} in Table § and the first trial vectors, V_{a1}

U_{a1}	1.68	4.52	4.14	4.12	4.16	3.34
$(U_{a1})^2$	2.8224	20.4304	17.1396	16.9744	17.3056	11.1556
V_{a1}	0.095	0.257	0.235	0.234	0.236	0.2

Step 3: Produce the second trial vector, V_{a2}

The elements of V_{a1} are “accumulatively multiplied” (Kline, 2002) by the first row of R to obtain the first element in a new vector, U_{a2} ; and then by the second row to obtain the second element; third row for the third element; *et cetera* (Table 2.).

$$0.095 \times 1 + 0.257 \times 0.42 + 0.235 \times 0.01 + 0.234 \times 0.45 + 0.236 \times 0.11 + 0.2 \times -0.31 = 0.325$$

$$0.095 \times 0.42 + 0.257 \times 1 + 0.235 \times 0.77 + 0.234 \times 0.96 + 0.236 \times 0.75 + 0.2 \times 0.62 = 0.920$$

$$0.095 \times 0.01 + 0.257 \times 0.77 + 0.235 \times 1 + 0.234 \times 0.62 + 0.236 \times 0.98 + 0.2 \times 0.76 = 0.950$$

$$0.095 \times 0.45 + 0.257 \times 0.96 + 0.235 \times 0.62 + 0.234 \times 1 + 0.236 \times 0.57 + 0.2 \times 0.52 = 0.902$$

$$0.095 \times 0.11 + 0.257 \times 0.75 + 0.235 \times 0.98 + 0.234 \times 0.57 + 0.236 \times 1 + 0.2 \times 0.75 = 0.938$$

$$0.095 \times -0.31 + 0.257 \times 0.62 + 0.235 \times 0.76 + 0.234 \times 0.52 + 0.236 \times 0.75 + 0.2 \times 1 = 0.795$$

Table 2. *Production of the second trial vector, V_{a2}*

U_{a2}	0.325	0.920	0.950	0.902	0.938	0.795
$(U_{a2})^2$	0.106	0.846	0.903	0.814	0.880	0.632
V_{a2}	0.159	0.450	0.465	0.441	0.459	0.389

Step 5. Produce the third trial vector

This is done as for the second (Table 3.).

$$0.159 \times 1 + 0.450 \times 0.42 + 0.465 \times 0.01 + 0.441 \times 0.45 + 0.459 \times 0.11 + 0.389 \times -0.31 = 0.481$$

$$0.159 \times 0.42 + 0.450 \times 1 + 0.465 \times 0.77 + 0.441 \times 0.96 + 0.459 \times 0.75 + 0.389 \times 0.62 = 1.884$$

$$0.159 \times 0.01 + 0.450 \times 0.77 + 0.465 \times 1 + 0.441 \times 0.62 + 0.459 \times 0.98 + 0.389 \times 0.76 = 1.832$$

$$0.159 \times 0.45 + 0.450 \times 0.96 + 0.465 \times 0.62 + 0.441 \times 1 + 0.459 \times 0.57 + 0.389 \times 0.52 = 1.865$$

$$0.159 \times 0.11 + 0.450 \times 0.75 + 0.465 \times 0.98 + 0.441 \times 0.57 + 0.459 \times 1 + 0.389 \times 0.75 = 1.813$$

$$0.159 \times -0.31 + 0.450 \times 0.62 + 0.465 \times 0.76 + 0.441 \times 0.52 + 0.459 \times 0.75 + 0.389 \times 1 = 1.746$$

Table 3. *Production of the third trial vector, V_{a3}*

U_{a3}	0.481	1.884	1.832	1.865	1.813	1.746
$(U_{a3})^2$	0.232	3.548	3.356	3.478	3.287	3.049
V_{a3}	0.076	0.765	1.103	1.143	1.08	1.002

Step 6. Produce further trial vectors

This process is continued until vectors are arrived at that are almost similar to the previous set; a process known as convergence. If a hypothetical vector V_{ag} was found to be almost identical to V_{af} , then V_{ag} becomes the first characteristic vector of the matrix. U_{ag} is squared and summed and then square-rooted. This square-root equals the first characteristic root or eigenvalue (λ). This process can be continued until as many components as variables have been calculated. However, the first few principal components usually account for most of the variation in the variables.

Step 7. Obtain the second principal component

The second principal component is calculated in exactly the same way as the first.

The result of this is to find the eigenvalues from which the eigenvectors are extracted.

Table 4. outlines the computer's result from the data above.

Table 4. *Eigenvalues, Percent Variances of the Eigenvalues and Eigenvectors*

EIGENVALUES PERCENT VALUE OF THE EIGENVALUES		3.96	1.45	0.43
		66.05	24.16	7.23
		EIGENVECTORS		
a.	Horse	0.11	0.78	0.31
b.	Donkey/ass	0.47	0.23	-0.24
c.	Mule	0.46	-0.19	0.39
d.	Zebra	0.43	0.31	0.54
e.	Goat	0.46	-0.15	0.55
f.	Pony	0.40	-0.43	-0.32

These were then plotted manually on a graph producing a concept map of a particular student's semantic space *sensu* George Kelly (1953). Fransella *et al.* (2004) state that the loadings are “the extent to which each construct relates to that component”, and they believe that it is not necessary to spend much time studying the loadings since it is these that allow the practitioner to draw a Cartesian plane (Figure 1)..

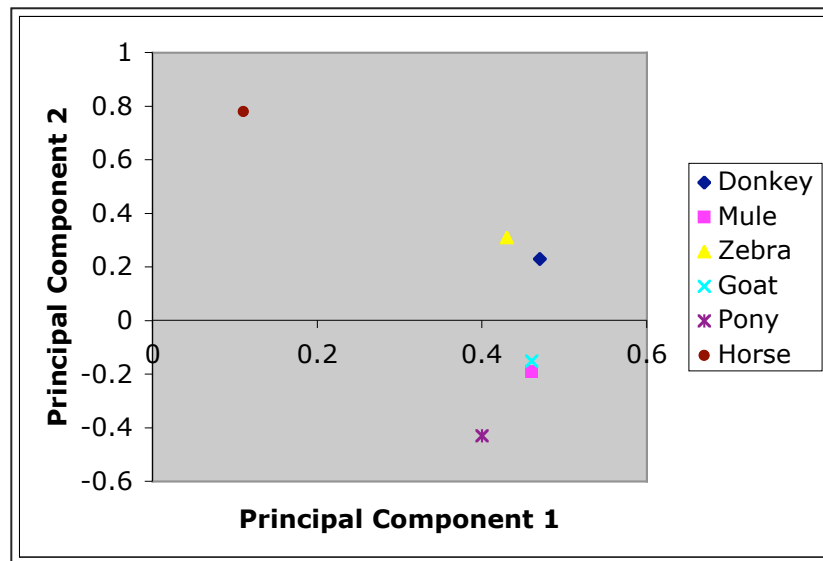


Figure 1. Plot of the First and Second Principal Axes Loadings (Components) as a scatter-graph, drawn in MS EXCEL™.